Solving Systems of Linear Equations

Key Points:

- A system of linear equations consists of two or more equations made up of two or more variables such that all equations in the system are considered simultaneously.

  For example, $\begin{aligned} 2x + y &= 15 \\ 3x - y &= 5 \end{aligned}$ is a system of linear equations in two variables.

- The solution to a system of linear equations in two variables is any ordered pair that satisfies each equation independently.

  In the above example, the order pair $(4,7)$ is the **solution** to the system of equations

$$2(4) + 7 = 15$$
$$3(4) - 7 = 5$$

- There are three types of systems of linear equations in two variables, and three types of solutions:

  - An **Independent System** has exactly one solution pair (x, y) The point where the two lines intersect is the only solution.
  - An **Inconsistent System** has no solution. Notice that the two lines are parallel and will never intersect.
  - A **Dependent System** has infinitely many solutions. The lines are coincident. They are the same line, so every coordinate pair on the line is a solution to both equations.

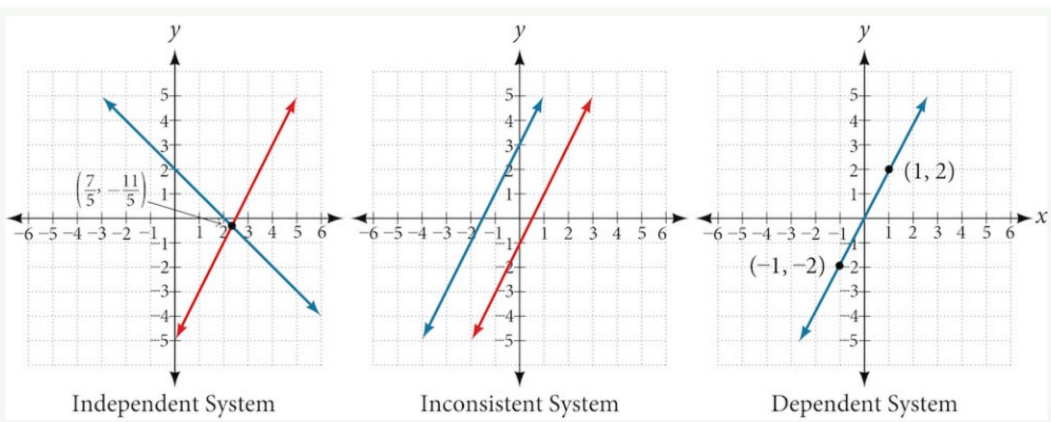

Figure 2

- One method of solving a system of linear equations in two variables is by graphing. In this method, we graph the equations on the same set of axes.

- Another method of solving a system of linear equations is by substitution. In this method, we solve for one variable in one equation and substitute the result into the second equation.

- A third method of solving a system of linear equations is by elimination, in which we can eliminate a variable by adding opposite coefficients of corresponding variables.

Solving Systems of Linear Equations Video

- **Determining whether an Ordered pair Is a Solution to a Systems of Equations**
- **Solving Systems of Linear Equations by Graphing**
- **Solving Systems of Linear Equations in Two Variables by Substitution**
- **Solving Systems of Linear Equations in Two Variables by the Elimination Method**
- **Identifying Inconsistent Systems of Equations Containing Two Variables**
- **Expressing the Solution of Systems of Dependent Equations Containing Two Variables**

Practice Exercises

Follow the instructions for each of the following exercises.

1. Determine whether the ordered pair is a solution to the system of equations:

$$\mathbf{3x - y = 4}$$
$$\mathbf{x + 4y = -3}$$
$$\mathbf{(-1, 1)}$$

2. Determine whether the ordered pair is a solution to the system of equations:

$$\mathbf{6x - 2y = 24}$$
$$\mathbf{-3x + 3y = 18}$$
$$\mathbf{(9, 15)}$$

3. Use substitution to solve the system of equations:

$$\mathbf{10x + 5y = -5}$$
$$\mathbf{3x - 2y = -12}$$

4. Use substitution to solve the system of equations:

$$\mathbf{\frac{4}{7}x + \frac{1}{5}y = \frac{43}{70}}$$
$$\mathbf{\frac{5}{6}x - \frac{1}{3}y = -\frac{2}{3}}$$

5. Use substitution to solve the system of equations:

$$\mathbf{5x + 6y = 14}$$
$$\mathbf{4x + 8y = 8}$$

6. Use elimination to solve the system of equations:

$$\mathbf{3x + 2y = -7}$$
$$\mathbf{2x + 4y = 6}$$

7. Use elimination to solve the system of equations:

$$\mathbf{3x + 4y = 2}$$
$$\mathbf{9x + 12y = 3}$$

8. Use elimination to solve the system of equations:

$$\mathbf{8x + 4y = 2}$$
$$\mathbf{6x - 5y = 0.7}$$

Answers:

1. No.

2. Yes.

3. $(-2, 3)$

4. $\left(-\frac{41}{75}, \frac{19}{30}\right)$

5. $(4, -1)$

6. $(-5, 4)$

7. No solutions exist.

8. $\left(\frac{\mathbf{1}}{\mathbf{5}}, \frac{\mathbf{1}}{\mathbf{10}}\right)$